Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–IV(OLD) – EXAMINATION – SUMMER 2019

Subject Code:140001 **Date:09/05/2019**

Subject Name: Mathematics-IV

Time:02:30 PM TO 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Find all roots of $\sqrt[3]{8i}$. **07**

(b) **04**

1) Find real and imaginary part of $f(z)=z^2+4z$. Also, calculate the value of f at $z=1+i$.

03

2) Show that $f(z)=\begin{cases}\dfrac{\mathrm{Im}(z)}{|z|}; & z\neq 0\\ 0; & z=0\end{cases}$ is not continuous at the origin.

Q.2 **(a)** Find the image of the region $|z|<1$ under the transformation $w=2z-i$. Sketch the region and its image. **07**

(b) Show that $u(x,y)=2x-x^3+3xy^2$ is harmonic in some domain D and find a harmonic conjugate of $u(x,y)$. **07**

OR

(b) If $f(z)$ is an analytic function of z, show that **07**

$$\left(\frac{\partial}{\partial x}|f(z)|\right)^2+\left(\frac{\partial}{\partial y}|f(z)|\right)^2=|f'(z)|^2$$

Q.3 **(a)** Evaluate $\int_0^{2+i} z^2\,dz$ along the line $y=x/2$ **07**

(b) Evaluate: **07**

1. $\oint_c \frac{z}{z-3}dz$, over the contour c, where c is the circle $|z|=1$.
2. $\oint_C \frac{e^z}{z(1-z)^3}dz$, counterclockwise over C, where C: $|z|=2$
3. $\oint_C \frac{e^z}{(z-1)(z-3)}dz$, counterclockwise over C, where C: $|z|=2$

OR

Q.3 **(a)** Determine the Laurent series expansion of $f(z)=\frac{1}{(z+1)(z+3)}$ valid for **07**

a) $|z|<1$ b) $1<|z|<3$

(b) Using Newton's divided difference formula, compute $f(10.5)$ from the following data: **07**

x:	10	11	13	17
f(x):	2.3026	2.3979	2.5649	2.8332

Q.4 **(a)** Find a real root of the equation $x^3+4x^2-1=0$, lies between 0 and 1 by using bisection method correct to decimal places. **07**

(b) Evaluate $\int_0^3 \frac{dx}{(1+x)}$ with n=6 by using Simpson's 3/8 rule and hence calculate ln 2. **07**

OR

Q.4 **(a)** Solve the following system of equation using partial pivoting by Gauss Elimination method. **07**

$$\begin{aligned} 8x_2 + 2x_3 &= -7 \\ 3x_1 + 5x_2 + 2x_3 &= 8 \\ 6x_1 + 2x_2 + 8x_3 &= 26 \end{aligned}$$

(b) Solve the following system of equations by using Gauss-Seidel method. **07**

$10x + y + z = 6;\ \ x + 10y + z = 6;\ \ x + y + 10z = 6$

Q.5 **(a)** Using the power method, find the largest eigenvalue of the matrix **07**

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

(b) Apply Runge-Kutta fourth order method to find an approximation value of y when x=0.1 in step of 0.1 if $\frac{dy}{dx} = x + y^2,\ \ y(0) = 1$ **07**

OR

Q.5 **(a)** Evaluate the integral $\int_0^1 \frac{dx}{(1+x)}$, by Gauss three point quadrature formula. **07**

(b) Solve the differential equation $\frac{dy}{dx} + xy = 0;\ y(0) = 1,$ from $x = 0$ to $x = 0.25$ using Euler's method taking step size 0.05. **07**

****